# CIEG 305 PROBLEM FORMULATIONS IN Ψ or Φ

If you had the assumptions below and wanted to try to solve a fluid mechanics problem for the flow field and pressure, you could approach in this fashion. You would also likely need some information about boundary conditions. Sound like this is a useless approach? Linear wave theory, which is used by many an oceanographer and coastal engineer can be developed from this methodology.

**PROBLEM FORMULATION IN Φ**

Assume inviscid $(\mu = 0)$, steady $\left(\frac{d}{dt} \to 0\right)$, incompressible $(\nabla \cdot \vec{V} = 0)$, irrotational $(\nabla \times \vec{V} = 0)$.

Then

$u = \frac{\partial \phi}{\partial x}, v = \frac{\partial \phi}{\partial y}$ and from continuity $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$$\Rightarrow \quad \frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial y}\right) = 0$$

or $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad also \quad written \quad as \quad \nabla^2 \phi = 0$

AHHHH! A linear equation because power of Φ =1.
Known as LaPlace Equation, a 2nd order PDE.

<u>Formulation</u>

Two Equations: LaPlace equation (incompressible, irrotational part) and Bernoulli equation (inviscid, steady part).

Two unknowns Φ, P (P is generally what we really need to solve for)

<u>Solution Method</u>

Solve LaPlace for Φ, Find u,v from definition of velocity potential. Determine P from Bernoulli equation.

**PROBLEM FORMULATION IN Ψ**

Assume inviscid $(\mu = 0)$, steady$\left(\frac{d}{dt} \to 0\right)$, incompressible$\left(\nabla \cdot \vec{V} = 0\right)$, irrotational $\left(\nabla \times \vec{V} = 0\right)$.

Stream function satisfies incompressible continuity constraint.

$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$ and from irrotational flow part $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$

$$\Rightarrow \quad \frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right) = 0$$

or $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \qquad \textit{also written as} \quad \nabla^2 \psi = 0$

LaPlace equation in Ψ.

Formulation

Two Equations: LaPlace equation (incompressible, irrotational part) and Bernoulli equation (inviscid, steady part).

Two unknowns Ψ, P (P is generally what we really need to solve for)

Solution Method

Solve LaPlace for V, Find u,v from definition of stream function. Determine P from Bernoulli equation.