

DERIVATION OF NEWTON'S LAW

Recall, for a solid, shear stress is proportional to strain (Hooke's Law)

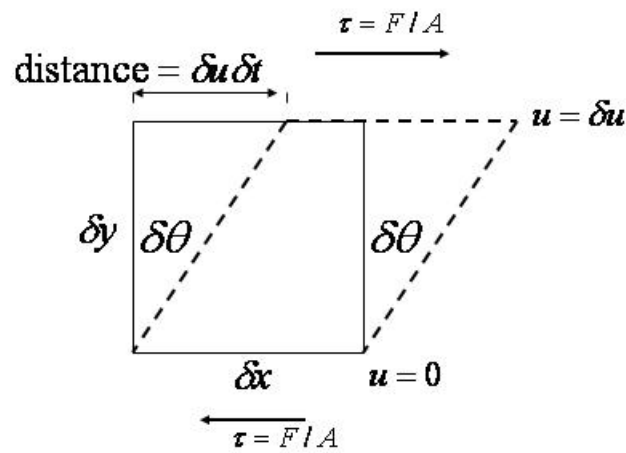
$$\tau = G\Gamma$$

$$\tau = F / A = \text{shear stress}$$

$$\Gamma = \text{shear strain}$$

$$G = \text{Elasticity modulus}$$

A fluid cannot sustain a shear stress, thus it must deform continuously under applied shear.

SHEARING OF A FLUID ELEMENT

From Geometry

$$\tan(\delta\theta) = \frac{\delta u \delta t}{\delta y}$$

take limit as $\delta\theta \rightarrow 0$

$$\delta\theta = \frac{\delta u \delta t}{\delta y}; \quad \tan(\theta) \approx \theta \quad \text{as } \theta \rightarrow 0$$

$$\text{strain rate} = \frac{\delta\theta}{\delta t} = \frac{\delta u}{\delta y}$$

The shear stress is assumed to be linearly related to the strain rate as

$$\tau \propto \frac{\delta u}{\delta y}$$

$$\tau = \mu \frac{\delta u}{\delta y}; \mu \text{ is the constant of porportionality (the molecular or absolute viscosity)}$$

Fluids that follow this relationship are known as Newtonian fluids (The kind we will be concerned with in this class).