Flow in a single pipe is characterized by 7 parameters:

Fluid: density (ρ), viscosity (μ)

Pipe: Diameter (D), Length (L), roughness (ε)

Flow: Flow rate/Velocity (Q or V) and Head loss/Pressure drop (h_f or ΔP)

3 types of problems are identified depending on which of the seven variables are known (**assuming no minor losses due separation and turbulence associated with pipe bends, pumps, entrances etc**)

Type	Given Information			Find	Method
	Fluid	Pipe	Flow		
I	P, μ	D, L, ε	Q (or V)	h_f or (ΔP)	Direct
II	P, μ	D, L, ε	h_f (or ΔP)	Q (or V)	Iterative
III	P, μ	L, ε	Q , h_f	D	Iterative

1. TYPE I problems (given ρ, μ, D, L, ε, Q; find h_f)

a) calculate the velocity, V, from continuity equation V=Q/A.

b) Calculate the Reynolds Number Re= ρVD/ μ and ε/D.

c) Find friction factor, f, from Moody diagram.

d) Calculate the frictional head loss from the Darcy-Weisbach equation

$$h_f = f \frac{L}{D} \frac{V^2}{2g}.$$

e) Find pressure drop if necessary from the energy equation.

2. TYPE II problems (given ρ, μ, D, L, ε, h_f ; find Q or V), note f is unknown

a) Write steady energy equation (including pumps and turbines where applicable):

$$\left(\frac{P}{\gamma}+\frac{V^2}{2g}+z\right)_1=\left(\frac{P}{\gamma}+\frac{V^2}{2g}+z\right)_2+h_f+h_t-h_p$$

b) Specify losses in terms of physical parameters of the problem (e.g. pressure head difference/elevation difference) -> gives h_f.

c) Define losses by Darcy-Weisbach equation.

$$h_f=f\frac{L}{D}\frac{V^2}{2g}.$$

d) Iterative solution

- Find ε/D
- Assume an f
- Calculate V from Darcy Weisbach equation
- Calculate Reynolds Number: $Re=\rho VD/\mu$.
- Find f from Moody diagram
- Is f the same (within 2 significant figures) as assumed f?
- If yes, stop and find Q=VA if necessary. If no, use this f as the assumed f and repeat steps 3-7 again.

Note: it is often useful to first assume a friction factor corresponding to fully turbulkent flow for the given values of ε/D since most flows are turbulent (and curve on Moody diagram is flat).

3.TYPE III problems (given ρ, μ, L, ε, h_f , Q; find D), note f is unknown

a) Define V from continuity equation: $V=Q/A = 4Q/\pi D^2$.

b) Write steady energy equation (including pumps and turbines where applicable):
$$\left(\frac{P}{\gamma}+\frac{V^2}{2g}+z\right)_1=\left(\frac{P}{\gamma}+\frac{V^2}{2g}+z\right)_2+h_f+h_t-h_p\,.$$

c) Use energy equation to define h_f.

d) Write Darcy Weisbach equation $h_f = f\,\frac{L}{D}\frac{V^2}{2g}$.

e) Combine with the continuity equation to get $h_f = f\,\frac{L}{D}\left(\frac{4Q}{\pi D^2}\right)^2\frac{1}{2g}=\frac{8fL}{g\pi^2}\frac{Q^2}{D^5}$.

f) Solve for D $D=\left(\frac{8LQ^2}{h_f g\pi^2}\right)^{1/5} f^{1/5}$.

g) Define Reynolds Number through the continuity equation: $\mathrm{Re}=\frac{\rho VD}{\mu}=\left(\frac{4Q\rho}{\pi\mu}\right)\frac{1}{D}$.

h) Iterative Solution.

- Assume f
- Calculate Diameter D from f
- Calculate Reynolds Number from g
- Calculate ε/D
- Find f from Moody diagram
- Is f the same (within 2 significant figures) as assumed f?
- If yes, stop. If no, use this f as the assumed f and repeat steps 2-7 again.

Note: Choose f in typical range of 0.02 – 0.03. Convergence will be fairly rapid due to exponent on f term.

Identification of pipe flow problems (some example phrases)

Type I (given ρ, μ, D, L, ε, Q; find h_f)
"determine elevation difference between 2 reservoirs"
"find pressure drop along pipe"
"will water leak from the pipe"
"find the maximum length of pipe permissible …"
"evaluate the loss coefficient .."
"what is the maximum elevation the siphon can reach at this flow without cavitating"
"find the necessary pump power to convey a flow of …"
"how much power is produced in the turbine when the flow rate is ..

Type II (given ρ, μ, D, L, ε, h_f ; find Q or V)
"Find the flow rate for the given pressure drop"
"what is the maximum flow rate possible without cavitation"
"find the velocity in the pipe .."
"how long will it take to fill a tank/pool"

Type III (given ρ, μ, L, ε, h_f , Q; find D)
" find diameter necessary to carry of flow of ___ with a pressure drop of ___"
" what size pipe is required to .."