CIEG 305 HANDOUT ON POTENTIAL FLOW SOLUTIONS

Type	Velocity	Stream Function	Velocity Potential
Uniform Stream in x direction	$u = U_\infty = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$ $v = 0 = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$ $v_r = U_\infty = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{\partial \phi}{\partial r}$ (*this is u*) $v_\theta = 0 = -\frac{\partial \psi}{\partial r} = \frac{1}{r}\frac{\partial \phi}{\partial \theta}$ (*this is v*)	$\psi = Uy$ $\psi = U_\infty r \sin\theta$	$\phi = Ux$ $\phi = U_\infty r \cos\theta$
Line source or sink at origin	$v_r = \frac{Q}{2\pi r b} = \frac{m}{r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{\partial \phi}{\partial r}$ $v_\theta = 0 = -\frac{\partial \psi}{\partial r} = \frac{1}{r}\frac{\partial \phi}{\partial \theta}$	$\psi = m\theta$	$\phi = m \ln r$
Line irrotational vortex	$v_r = 0 = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{\partial \phi}{\partial r}$ $v_\theta = \frac{K}{r} = -\frac{\partial \psi}{\partial r} = \frac{1}{r}\frac{\partial \phi}{\partial \theta}$	$\psi = -K \ln r$	$\phi = K\theta$

Where m =(Q/2πb) is a constant source/sink strength, b is length into paper, Q is volume flow rate.

K is the strength of the vortex (length squared per time).

One can go from polar to Cartesian and back using

$$x = r\cos\theta \quad y = r\sin\theta \quad r = \sqrt{x^2 + y^2} \quad \theta = \tan^{-1}\frac{y}{x}$$