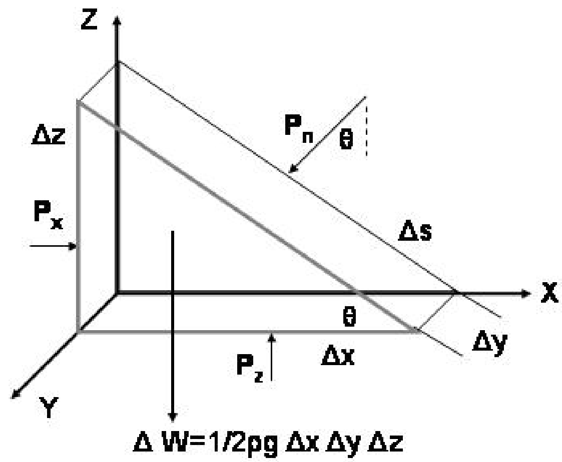
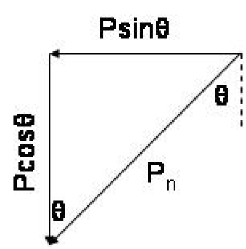
Pressure at a Point in a Static Fluid is Independent of Orientation

Consider a small wedge of fluid at rest



Postulate that P_x, P_z, P_n are different.

Step 1: Break pressure force vector acting on Δs into horizontal and vertical components



$\left|\vec{P}_n\right| = P$

Step 2: Horizontal Force Balance

$\sum F_H = 0$, since fluid at rest; no acceleration.

$P_x \Delta y \Delta z - P_n \sin(\theta) \Delta y \Delta s = 0$ P=F/A so F=PA.

$P_x \Delta z = P \sin(\theta) \Delta s$

But from geometry $\Delta z = \Delta s \sin(\theta)$

so $P_x \Delta s \sin(\theta) = P_n \sin(\theta) \Delta s$

or $P_x = P_n$

Step 3: Vertical Force Balance

$$\sum F_V = 0$$

$0 = P_z \Delta x \Delta y - P_n \cos(\theta) \Delta s \Delta y - \frac{1}{2} \rho g \Delta x \Delta y \Delta z$; last term is weight of wedge

But from geometry $\Delta x = \Delta s \cos(\theta)$

$$0 = P_z \Delta s \cos(\theta) - P_n \cos(\theta) \Delta s \Delta y - \frac{1}{2} \rho g \Delta s \cos(\theta) \Delta y \Delta z$$

$$0 = P_z - P_n - \frac{1}{2} \rho g \Delta z$$

or $P_z = P_n + \frac{1}{2} \rho g \Delta z$

TWO IMPORTANT POINTS:

1) No pressure change in horizontal direction for fluid at rest.
2) Pressure in vertical direction is proportional to density, gravity and depth.

Now take the limit as $\Delta z \to 0$

$$P_z = P_n$$

THUS, $P_z = P_n = P_x$

Since θ was arbitrary, implies pressure at a point in a static fluid is independent of orientation.