Proof: why does vertical force on curved surface equal weight of fluid above even when the fluid is below?

\[ F_{CD} = F_v + W_{BCD} \]

\[ F_v = F_{CD} - W_{BCD} \]

\[ W_{BCD} = \gamma Ab, \text{ } b \text{ equals distance in to paper} \]

\[ W_{BCD} = \gamma b \left( r^2 - \frac{\pi r^2}{4} \right), \text{ the square minus the quarter circle} \]

\[ F_{CD} = \gamma A = \gamma r(b) = \gamma br^2 \]

Thus

\[ F_v = \gamma br^2 - \gamma b \left( r^2 - \frac{\pi r^2}{4} \right) \]

\[ F_v = \gamma br^2 - \gamma br^2 + \gamma b \frac{\pi r^2}{4} \]

\[ F_v = \gamma b \frac{\pi r^2}{4}, \text{ this is the weight of the fluid above the surface even though it is not there!} \]