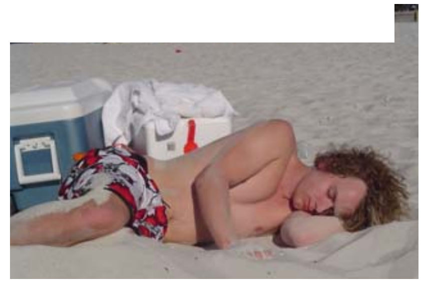
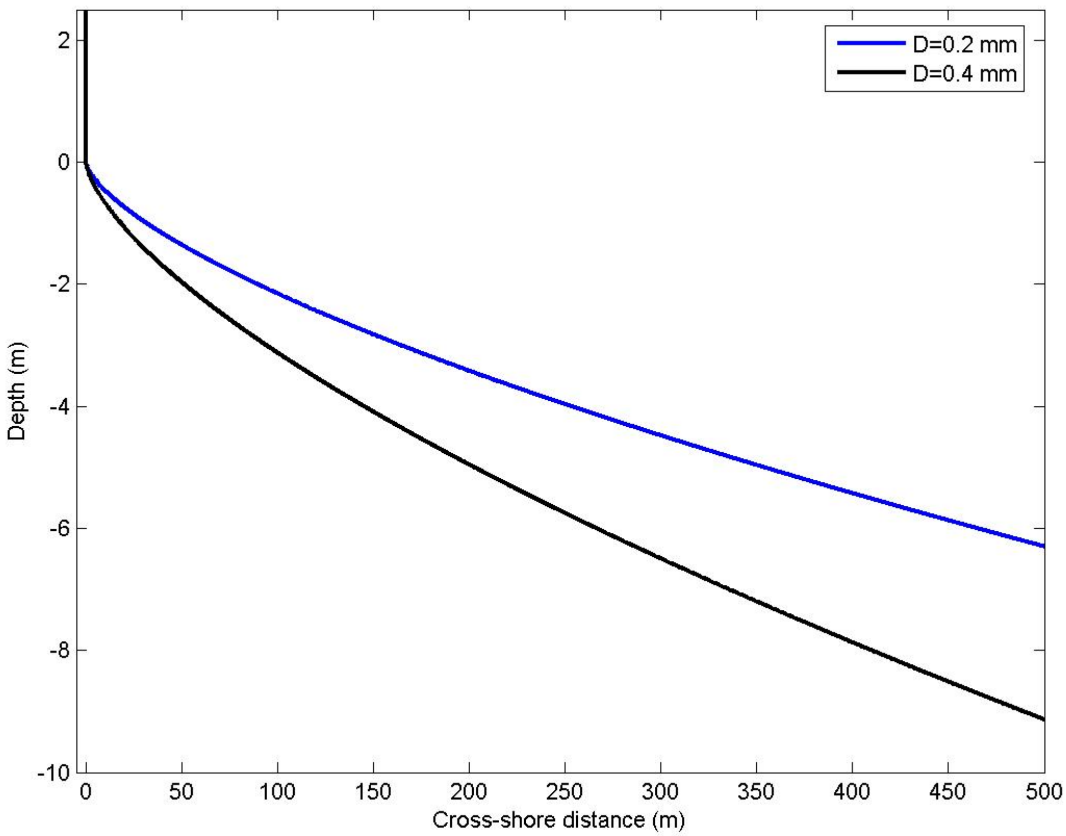
CIEG 680 Homework #5



1. DD 7.2

For the grain sizes given, A= 0.1 (for 0.2 mm) and A=0.145 (for 0.4 mm). Using matlab to plot the profiles from the simple EBP theory leaves



2. DD 7.13

Extraction causing the land to sink at 2mm/yr. If no nourishment carried out, what is effect on the shoreline position?

If the land is sinking, the water depth at each cross-shore position increases. The profile is out of equilibrium and the shoreline must retreat as sand is transported offshore.

Quantify any shoreline change rate

From the Bruun rule we have $\Delta y = \frac{-SW_*}{(h_* + B)}$.

For this problem, S = 2 mm/yr. W* is found from the equation for the equilibrium beach profile as $W_* = \left(\frac{h_*}{A}\right)^{3/2}$ = 464.8 m.

Thus the change rate is $\frac{2x10^{-3}(m/yr)464.8(m)}{8m} = 0.11(m/yr)$

What would be the annual nourishment requirements per unit of beach length in order to obtain a stable shoreline?

We must integrate the across the active profile the volume required to keep the profile relative to the new water level unchanged.

$\Delta V = W_* S$ or $\Delta V = 464.8(m) * 2x10^{-3}(m/yr) = 0.93(m^3 / mofbeach)$

NOT MUCH!

3. DD 7.16

Compare the beach recessions to be expected for sand sizes of 0.1 and 0.3 mm for the following storm conditions: H_b = 2 m, S = 1m, B= 2m. Use the simplest method (Bruun's) of the three available.

Bruun's rule : $\Delta y = \frac{-SW_*}{(h_* + B)}$.

Approach 1: From H_b = 2 m find $W_*=19.3H_b$ = 38.6 m from eqn 7.21. For sand sizes of 0.1 and 0.3 mm, A = 0.063 and 0.125 $m^{1/3}$ respectively. We find $h_* = AW_*^{2/3}$ or 0.72 m and 1.43 m respectively. Plugging these values in Bruun's rule, the recession is found to be 14.2m and 11.25

m respectively. Intuitively this appears to give W_* and h_* values that are too small for a 2.5 m wave

Approach 2: From H_b = 2 m find h_b = 2.5m from the spilling breaker assumption. For sand sizes of 0.1 and 0.3 mm, A = 0.063 and 0.125 $m^{1/3}$ respectively. Using the equation for the equilibrium beach profile, we find $W_* = \left(\frac{h_*}{A}\right)^{3/2}$ = 250 m and 89 m respectively. Plugging these values in Bruun's rule, the recession is found to be 55.6 and 19.8 m respectively.

4. DD 7.20

7.19. Bruun's rule equates the "volume required" by sea level rise to the volume "yielded" by shoreline retreat. Consider the situation shown in Figure 7.32 with a berm height B, a depth of closure h_* , and a layer of organic material (i.e. no sand) of thickness Δz.

(a) Develop a relationship for the ratio R/S (shoreline retreat/sea level rise), including the effect of the organic layer thickness Δz.

Following the same procedure as for the development of Bruun's rule. The volume required is

$$\Delta V_- = W_* S$$

The volume generated is now reduced by the amount of the organic layer to be

$$\Delta V_+ = R(h_* + B) - R\Delta z \ .$$

Equating and solving for the ratio R/S

$$\frac{R}{S} = \frac{W_*}{h_* + B - \Delta z} .$$

(b) Calculate the shoreline retreat for the following conditions:

B = 2m
h_* = 8m
S = 0.1m
A = 0.1 $m^{1/3}$

Carry out these calculations for Δz = 0 and Δz = 2m. Discuss the differences.

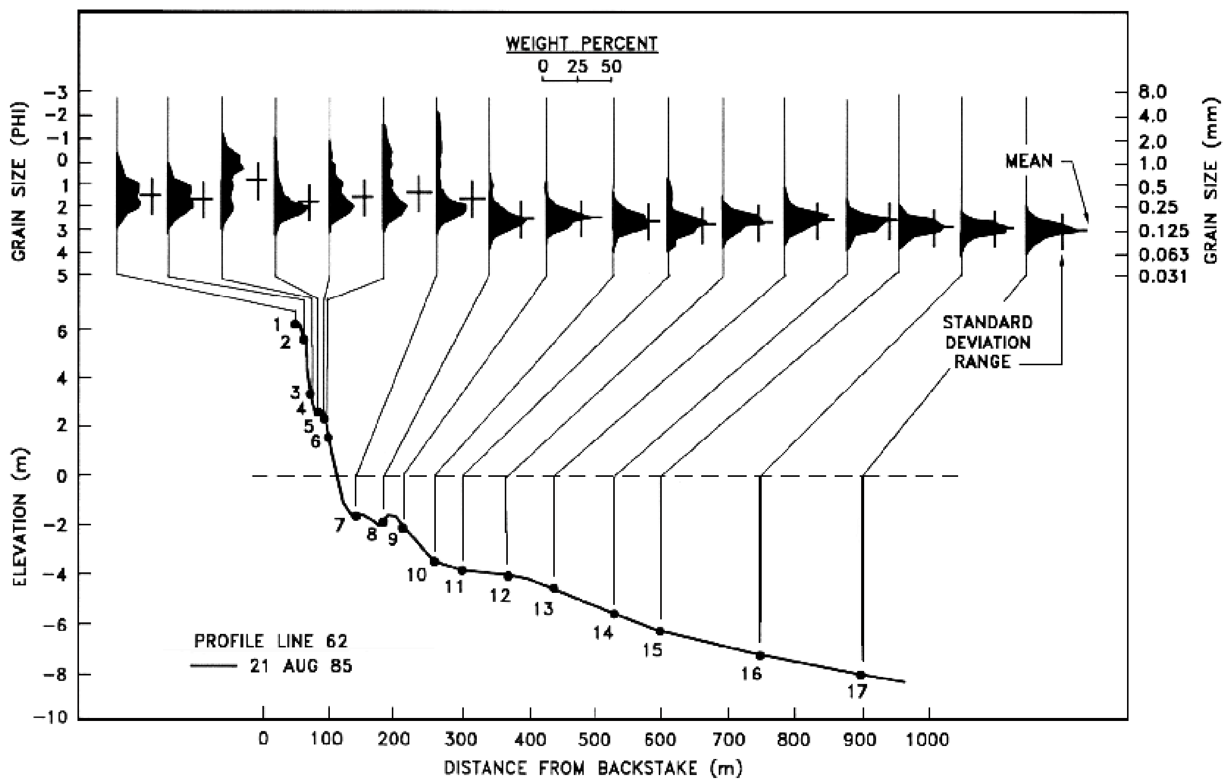
W* is given by $\left(\frac{h_*}{A}\right)^{3/2} = \left(\frac{8}{0.1}\right)^{3/2} = 715.5m$

For Δz = 0, the shoreline retreat is $R = \frac{W_* S}{h_* + B - \Delta z} = \frac{715.5 * 0.1}{8 + 2 + 0} = 7.15m$

For Δz = 2m, the shoreline retreat is $R = \frac{W_* S}{h_* + B - \Delta z} = \frac{715.5 * 0.1}{8 + 2 - 2} = 8.94m$.

As the thickness of the layer increases, the amount of shoreline recession increases since the recession has to account for the lack of sediment volume occupied by the organic layer.

5. Using the picture below from Stauble (1992), extract mean sediment grain sizes at the locations shown. Then go to www.frf.usace.army.mil and find the CRAB data for this profile. Calculate and plot the underwater portion of the profile using EBP theory based on equations 7.31 and 7.33. You will need Table 7.2 to determine you're A values. MATLAB CODE ATTACHED



From Stauble, 1992

SOLUTION

The table below shows the data I extracted from the grain size dist and the profile

Pt #	X location	D_{50}, mm	Elevation, m	A $m^{1/3}$
6.5	100	0.43	0	.1498
7	130	0.37	-1.75	.139

8	195	0.23	-1.95	.109
9	215	0.23	-2.00	.109
10	260	0.18	-3.7	.0936
11	300	0.14	-3.9	.0798
12	370	0.14	-4.0	.0798
13	440	0.17	-4.25	.0904
14	525	0.17	-5.5	.0904
15	600	0.13	-6.1	.0756
16	750	0.13	-7.0	.0756
17	900	0.125	-8.0	.0735

There will be 2 ways to go about this. First, we can assume a constant A value from y_n to y_{n+1}. Here we can take that constant value as the mean A from each location. The equation to use here is 7.31 given as

$$h_{n+1} = \left(h_n^{\frac{3}{2}} + A_c^{\frac{3}{2}} \{ y_{n+1} - y_n \} \right)^{\frac{2}{3}}$$

, where A_c is the mean A value from the 2 locations, y_{n+1} and y_n.

Option 2 is to assume a linearly varying A value between y_n to y_{n+1} and the following equation, 7.33 gives (as long as $A_{n+1} \neq A_n$)

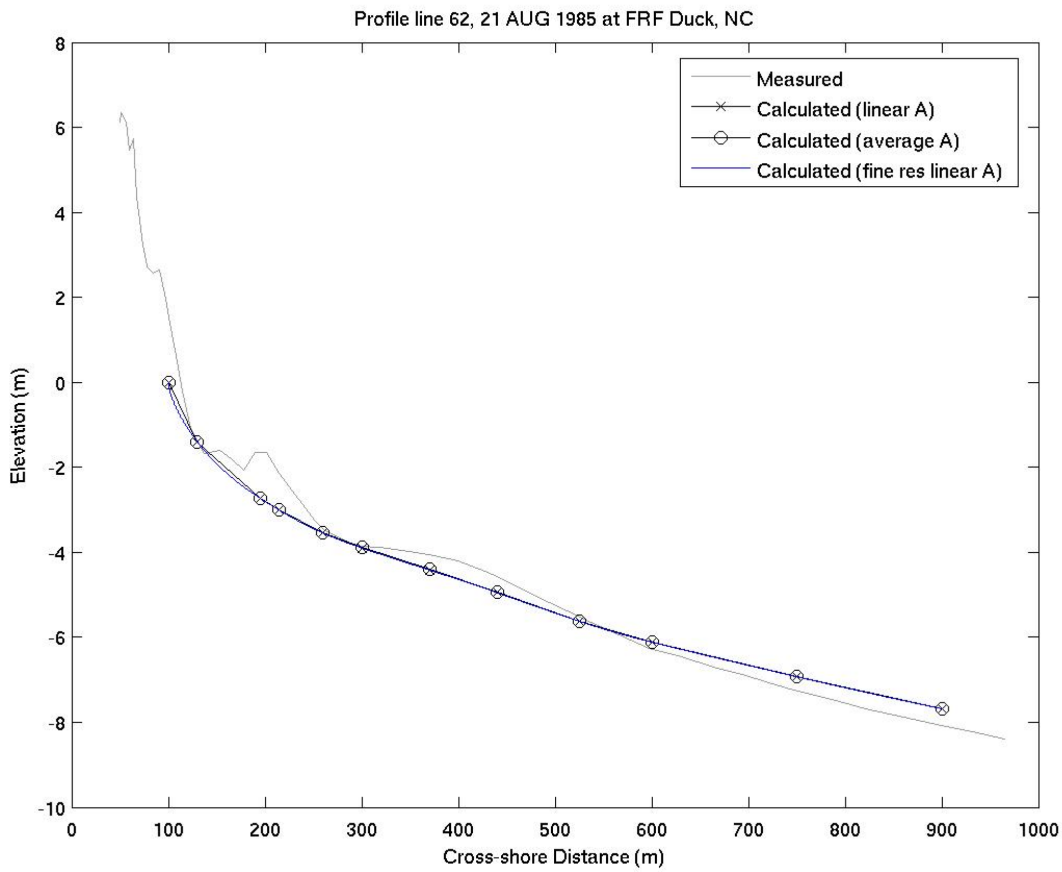
$$h_{n+1} = \left\{ h_n^{\frac{3}{2}} + \frac{2}{5m_n} \left[A_{n+1}^{\frac{5}{2}} - A_n^{\frac{5}{2}} \right] \right\}^{\frac{2}{3}}$$

where

$$m_n = \frac{A_{n+1} - A_n}{y_{n+1} - y_n}$$

, in other words, A varies linearly from y_n to y_{n+1}. If $A_{n+1} = A_n$ then we revert back to a constant A value and employ equation 7.31 in those cases.

The following figure shows the original profile and the 2 calculated profiles. Since the A values were often the same or varied little from location to location, the profiles are roughly equal but not exactly the same (can't really tell this from figure). In general, there is a fairly good agreement. Of course the EBP theory misses the sand bars, but we knew that was the case beforehand. There is slight under prediction of depth landward of the bars and in the deeper water, and clearly an over prediction directly at the sandbar locations. The shoreline I grabbed from the plot given is slightly landward of the actual shoreline, but does not affect the general shape of the EBP.



Profile line 62, 21 AUG 1985 at FRF Duck, NC
Measured
Calculated (linear A)
Calculated (average A)
Calculated (fine res linear A)
Elevation (m)
Cross-shore Distance (m)
8
6
4
2
0
-2
-4
-6
-8
-10
0
100
200
300
400
500
600
700
800
900
1000

```
% program for problem 5 from HW 5

% take data from the stauble 1992 figure for grain sizes at Duck, obtain A values and try to
estimate
% EBP assuming 1) linear variation in A between two known points and
               2) average A value between 2 known points


%% data extracted from plot ( I estimated the value at the SWL)

    loc=[100 130 195 215 260 300 370 440 525 600 750 900];  % location (y)
    hmeas=[0 1.75 1.95 2 3.7 3.9 4 4.25 5.5 6.1 7.0 8.0];   % known depth
    A=[.1498 .139 .109 .109 .0936 .0798 .0798 .0904 .0904 .0756 .0756 .0735];  % A from
table
                        %% see solutions for estimated grain diameters


    hest(1)=0;  % will be the linear one
    hest2(1)=0; % will be the average one

    for n=1:length(A)-1;
      Ac=(A(n+1)+A(n))/2;   % mean A value
        if A(n)~=A(n+1);  % if A's diff used linear A eqn
          m=(A(n+1)-A(n))/(loc(n+1)-loc(n));
          hest(n+1)=(hest(n)^1.5+2/(5*m)*(A(n+1)^2.5-A(n)^2.5))^(2/3); % linear A
          hest2(n+1)=(hest2(n)^1.5+Ac^1.5*(loc(n+1)-loc(n)))^(2/3);    % average A
        else  % A's are the same
          hest(n+1)=(hest(n)^1.5+Ac^1.5*(loc(n+1)-loc(n)))^(2/3); %linear=average here
          hest2(n+1)=(hest2(n)^1.5+Ac^1.5*(loc(n+1)-loc(n)))^(2/3);  % average A
        end
    end

    % make figure
    load /disk1/hw/litproc/d850821_duck.3d  % from FRF webpage
    dd=find(d850821_duck(:,2)==62);  % get just the profile line of interest
    h=plot(d850821_duck(dd,6),d850821_duck(dd,8),'k-');  % plot it
    set(h,'color',[.6 .6 .6]);  % change color
    hold on
    plot(loc,-hest,'k-x');  % plot linear A estimate
    plot(loc,-hest2,'k-o'); % plot average A estimate

    legend('Measured','Calculated (linear A)','Calculated (average A)');
    xlabel('Cross-shore Distance (m)');
    ylabel('Elevation (m)');
    title('Profile line 62, 21 AUG 1985 at FRF Duck, NC')
```

```
%%%%% a separate and more straightforward way to do it assuming linear variation in A
%% makes profile smoother too

ywant=100:0.1:900;  % where we want y values

Awant=interp1(loc,A,ywant);  %Do a linear interp to get the A values where we want them

hwant(1)=0;  % first value starts as SWL
for k=2:length(ywant)
 hwant(k)=( hwant(k-1)^1.5 + Awant(k-1)^1.5 * (ywant(k)-ywant(k-1))     )^(2/3);  % eqn 7.31 in D&D
end


  plot(ywant,-hwant,'r');  % to overlay the more highly resolved plot.  Note how this passes
      %through the other points but looks more curvaceously smooth.
```

6. **DD 7.4 (should be familiar)**

Why generally are coarser sediment found on the landward portion of the profile and finer sediment found seaward?

If coarseness is related to density, then the coarse sediment can withstand destructive forces better than fine sediment. The skewed nature of waves in the surf zone tends to preferentially push the coarser sediments landward. These sediments will likely reside near the beach step where shoreline wave energy is greatest. Finer sediments, are more easily mobilized and tend to stay in suspension longer. They cannot withstand destructive forces like coarser sediment can. Because they stay in suspension longer, they have a better chance of being transported offshore via gravity or undertow currents.

7. **Nielsen (1992) on page one states that " .. in general terms the boundary layer thickness obeys the formula**

$\delta = \sqrt{\nu_t T}$ **where T is the flow period.**

PROVE IT! Starting from the equation given below for unidirectional flow over a boundary, non-dimensionalize the equation and show that the constants in front of the pressure and x-diffusion term become very common non-dimensional numbers used in fluid mechanics. Based on the order of magnitude of these non-dimensional numbers you should be able to perform a scaling between two of the terms to determine the boundary layer thickness. Clearly the vertical-diffusion term must come into play since it is the only term that contains δ.

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu_t \frac{\partial^2 u}{\partial x^2} + \nu_t \frac{\partial^2 u}{\partial z^2}, \quad \text{assuming a constant eddy viscosity}$$

From the result, how much thicker would the boundary layer for a tidal flow be as compared to a typical wave?

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From the result, how much thicker would the boundary layer for a tidal flow be as compared to a typical wave?

Find a suitable non-dimensionalization such that we can determine the order of magnitude of the terms (which are important).

We must therefore scale all variables of interest as follows (with prime indicating a non-dimensional quantity)

$$x' = xk; \quad \text{k} = 1/(\text{wavelength of motion})$$

$$z' = \frac{z}{\delta}; \quad \delta \text{ is the boundary layer thickness}$$

$$t' = t\sigma; \quad \sigma \text{ is the angular frequency} = 1/\text{T}, \text{T} = \text{wave period}$$

$$u' = \frac{u}{a\sigma}; \quad \text{a} = \text{wave amplitude}$$

$$\text{P}' = \frac{\text{P}}{\rho \text{ga}}$$

Now we must redefine the derivatives based on the non-dimensional variables

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x'}\frac{\partial x'}{\partial x} = k\frac{\partial}{\partial x'}$$

$$\frac{\partial^2}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\right) = \frac{\partial}{\partial x}\left(k\frac{\partial}{\partial x'}\right) = \frac{\partial}{\partial x'}\left(k\frac{\partial}{\partial x'}\right)\frac{\partial x'}{\partial x} = k^2\frac{\partial^2}{\partial x'^2}$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t'}\frac{\partial t'}{\partial t} = \frac{\partial}{\partial t'}\sigma = \sigma\frac{\partial}{\partial t'}$$

$$\frac{\partial^2}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial}{\partial z}\right) = \frac{\partial}{\partial z}\left(\frac{\partial}{\partial z'}\frac{\partial z'}{\partial z}\right) = \frac{\partial}{\partial z}\left(\frac{\partial}{\partial z'}\frac{1}{\delta}\right) = \frac{\partial}{\partial z'}\left(\frac{1}{\delta}\frac{\partial}{\partial z'}\right)\frac{\partial z'}{\partial z} = \frac{1}{\delta^2}\frac{\partial^2}{\partial z'^2}$$

Now substitute into original equation

$$\frac{\partial}{\partial t'}(\sigma u' a\sigma) = -\frac{1}{\rho}k\frac{\partial}{\partial x'}(P'\rho g a) + \nu k^2\frac{\partial^2}{\partial x'^2}(u' a\sigma) + \frac{\nu}{\delta^2}\frac{\partial^2}{\partial z'^2}(u' a\sigma)$$

Collect constants outside derivatives

$$a\sigma^2\frac{\partial u'}{\partial t'} = -gak\frac{\partial P'}{\partial x'} + \nu k^2 a\sigma\frac{\partial^2 u'}{\partial x'^2} + \frac{\nu a\sigma}{\delta^2}\frac{\partial^2 u'}{\partial z'^2}$$

Divide through to isolate the time derivative

$$\frac{\partial u'}{\partial t'} = -\frac{gk}{\sigma^2}\frac{\partial P'}{\partial x'} + \frac{\nu k^2}{\sigma}\frac{\partial^2 u'}{\partial x'^2} + \frac{\nu}{\delta^2\sigma}\frac{\partial^2 u'}{\partial z'^2}$$

Now we look at constants in front of terms

For the pressure, the constant is the inverse of a Froude number which have values O(1).

For the x diffusion term the constant is the inverse of the Reynolds Number, which for typical flows is on the order of the inverse of 10^5 or 10^6. SO IT IS SMALL

Thus the constant on the vertical diffusion term must be O(1) to balance the other order 1 term. This implies

$\frac{\nu}{\delta^2\sigma} \approx 1 \quad or \quad \delta \approx \sqrt{\frac{\nu}{\sigma}}$. So we can get a feel for the expected magnitude of the boundary layer thickness.

For a tidal flow, the period is approximately 12 hours, whereas a typical wave may have a period of 10 seconds. For a constant eddy viscosity this ratio becomes $\sqrt{\frac{12*60*60}{10}} = 66$. So, tidal boundary layer is expected to be approximately 66 times thicker.

May or may not be true as the eddy viscosity associated with waves may be much larger than associated with tides?