Impact of a Jet – Conservation of Linear Momentum

- Measure the force on the plate $F_{ex}$ due to the jet (jet-vane system) via balance of moment (weight beam).
- Calculate the force on the plate $F_j$ using the linear momentum balance.
- Compare measured value $F_{ex}$ with theoretically derived value $F_j$. Discuss errors causing the observed difference between $F_{ex}$ and $F_j$.  

1. Calculate the force on the plate by the jet using linear momentum balance:

Use conservation of energy to obtain jet velocity impinging onto the plate:

\[ z_0 + \frac{P_0}{\gamma} + \frac{V_0^2}{2g} = z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + h_L \]

Neglect head loss; \( P_0 = P_1 = 0 \) atmosphere pressure; \( z_1 - z_0 = S \)

\[ \Rightarrow \frac{V_0^2}{2g} = S + \frac{V_1^2}{2g} \]

\[ \Rightarrow V_1 = \sqrt{V_0^2 - 2gS} \]

**Note**

\( V_0 = Q/A \) can be obtained from the measurement:

\( Q \): total measured volume in weight tank/duration
In a given direction:
net force = net momentum fluxes
Momentum flux = mass flux times velocity
mass flux = $\rho \vec{V} \cdot \vec{A}$

$\vec{V} \cdot \vec{A} \Rightarrow$ Positive  
$\vec{V} \cdot \vec{A} \Rightarrow$ Negative

⇒ Positive if flow is out of control surface (C.S.); Negative if flow is into C.S.

Conservation of linear momentum in $z$-direction:

$$\sum \limits_z F = \sum \limits_z \dot{m}_{out} V_{out} - \sum \limits_z \dot{m}_{in} V_{in}$$

$-\dot{m}_{in} V_1 = F_1, \quad F_1 = -F_j$

$\therefore F_j = \dot{m}_{in} V_1 = \rho Q V_1$

$F_1$: force exerted by the plate onto the jet
$F_j$: reactive force; force exert by the jet onto the plate
2. Jet force can be directly measured from the balance of moment of the weight-beam system:

\[ F_{sp} \cdot a = W \cdot g \cdot b \]

Note: given in class note \( W = 0.6 \) Kg

\[ F_{sp} \cdot a + 0.15_{(m)} \cdot F_{ex} = W \cdot g \cdot (b + x) \]

\( F_{ex} \) calculated this way can be compared with that calculated by linear momentum balance \( F_j \).
Error Analysis:

\[ F_j = \rho Q V_1 \]

Error in \( F_j \) can be caused by \( Q \) and \( V_1 \)

\[ F_j + \Delta F_j = \rho (Q + \Delta Q)(V_1 + \Delta V_1) \]

\[ F_j + \Delta F_j = \rho Q V_1 + \rho \cdot Q \cdot \Delta V_1 + \rho \cdot \Delta Q \cdot V_1 + \rho \cdot \Delta Q \cdot \Delta V_1 \]

Normalized by \( F_j \), we obtain:

\[ \frac{\Delta F_j}{F_j} = \frac{\Delta Q}{Q} + \frac{\Delta V_1}{V_1} \]

This relation is linear.

i.e., 2% error in \( Q \) and 3% error in \( V_1 \) make total error in estimating \( F_j \) to be 5%.

\[ F_j = \frac{W g x}{0.15} \]

Error in \( F_j \) can be caused by \( x \) and \( W \)

\[ F_j + \Delta F_j = (x + \Delta x)(W + \Delta W) \frac{g}{0.15} \]

\[ = xW \frac{g}{0.15} \]

\[ + \frac{g}{0.15} (x \cdot \Delta W + \Delta x \cdot W + \Delta x \cdot \Delta W) \]

\[ \therefore \Delta F_j = \frac{g}{0.15} (x \cdot \Delta W + \Delta x \cdot W) \]

\[ \Rightarrow \frac{\Delta F_j}{F_j} = \frac{\Delta W}{W} + \frac{\Delta x}{x} \]
For a circular plate:

1. Energy balance:

\[
\begin{align*}
z_0 + \frac{P_0}{\gamma} + \frac{V_0^2}{2g} &= z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + h_L \\
\Rightarrow \frac{V_0^2}{2g} &= S + \frac{V_1^2}{2g} \\
\Rightarrow V_1 &= \sqrt{V_0^2 - 2gS}
\end{align*}
\]

2. Momentum balance:

\[
\sum_{z} F = -\dot{m}_{in} V_1 + 2\dot{m}_{out}(-V_1) = F_1
\]

Mass conservation suggest:

\[
\dot{m}_{in} = 2\dot{m}_{out}
\]

\[
\Rightarrow F_1 = -2\dot{m}_{in} V_1 = -F_j
\]

Force for circular plate is two times larger than that of flat plate!