Stability of a Floating Body
- Applications to Naval Architecture & Ocean Eng.

1. Inclination test
2. Stability test
Inclination Test

Apply a load that cause the barge to tilt (move $W_h$ and $W_v$ in x- and z-direction); Measure the angle of tilt; Check if the measured tilt angle matches with theoretical value; If not completely matching, discuss errors ...

When $\theta=0$:
Center of gravity $G$ and Buoyancy center $B$ of the barge are both in-line with z-axis and there is no net moment w.r.t. origin $O$.

When moving $W_H$ to the right of the origin, you introduce a load that gives a net moment w.r.t. origin $O$. The barge must adjust itself to balance that net moment. This causes the barge to tilt with an angle $\theta$.

d is the submerged depth when $\theta=0$
B is somewhere below G
Force balance requires:
\[ F_w = F_B \]

Moment balance requires:
\[ \sum M_o = 0 \]

Define counterclockwise as positive

\( F_w \) is the body force acting through the center of gravity \( G \)

\( F_B \) is buoyancy force acting through buoyancy center \( B \)
\[ \sum M_O = 0 \]

\[ \Rightarrow -\left( F_w \cos \theta x_G + F_w \sin \theta z_G \right) + \left( F_B \cos \theta x_B + F_B \sin \theta z_B \right) = 0 \]

Because \( F_w = F_B \)

\[ \Rightarrow -\left( x_G \cos \theta + z_G \sin \theta \right) + \left( x_B \cos \theta + z_B \sin \theta \right) = 0 \]

\( x_G, z_G, x_B, z_B \) are given in lab handout

\[ x_B = \frac{b^2 \tan \theta}{12d} \quad z_B = \frac{b^2 \tan^2 \theta}{24d} + \frac{d}{2} \]  

(related to the shape of the immersed volume)

\[ x_G = \frac{W_h x_h}{W} \quad z_G = \frac{W_B z_B + W_v z_v + W_h z_h}{W} \]  

(depends on the location of weights)

Weight of barge \( W_b, z_b = 5.2 \) cm

\( x_h \) and \( z_v \) change every run as you move the weights

Remember to measure \( z_h \)

\[ \Rightarrow \frac{b^2}{24d} \left( \tan \theta \right)^3 + \left( \frac{b^2}{12d} + \frac{d}{2} - z_G \right) \tan \theta - x_G = 0 \]

Algebraic equation for \( \tan \theta \). Find \( \tan \theta \) using Matlab function “roots”
**Stability Test**

$F_w$, the weight of floating body gives “destabilizing” moment, clockwise, makes $\theta$ larger.

$F_B$, the buoyancy force gives a “stabilizing” moment, counterclockwise, make $\theta$ smaller.

During the experiment, you will remove $W_H$ and put it on the mast ($z$-axis) with $W_v$. As you move $W_v$ (or $W_H$) higher, $z_G$ increases. For a given small angle of perturbation, increased $z_G$ gives larger destabilizing moment. If this is too large, the stabilizing moment due to $F_B$ cannot bring the barge back!
Previously from balance of moment:

\[ \sum M_0 = \frac{b^2}{24d} (\tan \theta)^3 + \left( \frac{b^2}{12d} + \frac{d}{2} - z_G \right) \tan \theta \]

with \( x_G = 0 \) in this case

We need \( \sum M_0 > 0 \) (counter-clockwise) so that restoring force due to buoyancy is greater than destabilize for due to gravitational weight.

For small \( \theta \), \( (\tan \theta)^3 \ll \tan \theta \)

\[ \left( \frac{b^2}{12d} + \frac{d}{2} - z_G \right) \tan \theta > 0 \]

\[ \therefore z_G < \frac{b^2}{12d} + \frac{d}{2} \] for stable condition.

i.e., for a given barge, \( \frac{b^2}{12d} + \frac{d}{2} \) is fixed. If we distribute the weight more to the bottom of the barge, the system is more stable.