Flow from a Hole

Part I:
Measure the trajectory of the jet flow in order to determine velocity leaving the hole.
Test measure results with Bernoulli equation and conservation of energy;
Get the velocity loss coefficient $C_v$.

Part II:
Measure flow discharge and get $C_d$ based on constant head exp. (Part I) and variable head exp. (Part II).
Understand “vena contracta”.

Part III:
In your report, compare $C_v$ and $C_d$ you obtained with text book results.
To get exit flow velocity at ②, use Bernoulli equation. Note: Energy loss is neglected here because Bernoulli equation is derived based on inviscid, steady, incompressible flow assumptions.

Apply Bernoulli equation from ① \rightarrow ②

\[ p_1 + \frac{1}{2} \rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2} \rho V_2^2 + \gamma z_2 \]

\( \gamma = \rho g \): specific gravity of water

\[ V_2^2 = \frac{2\gamma}{\rho} (z_1 - z_2) \]

\[ \Rightarrow V_2 = \sqrt{2gh} \]
In reality, there shall be some energy loss. Hence, we shall use conservation of energy:

Apply conservation of energy from ① → ②:

\[
\frac{P_1^0}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2^0}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L
\]

\( h_L \): head loss

\[
V_2 = \sqrt{2g(h - h_L)}
\]

\[
= \sqrt{2gh \left(1 - \sqrt{\frac{h_L}{h}}\right)}
\]

define: \( C_v = 1 - \sqrt{\frac{h_L}{h}} \)

\[
\Rightarrow V_2 = C_v \sqrt{2gh}
\]

\( C_v \) is determined empirically
Consider an object (a control volume of water) with $x$-direction velocity $V_2$ but in the mean time, also falling due to gravity:

$$x = V_2 t$$
$$y = \frac{1}{2} gt^2$$

To eliminate $t$, use $t = \frac{x}{V_2}$

$$\Rightarrow y = \frac{1}{2} g \frac{x^2}{V^2}$$

previously: $V_2 = C_v \sqrt{2gh}$

$$\Rightarrow x = C_v \sqrt{4yh}$$

During experiment, measure trajectory to get $x_i$, $y_i$. Then plot $x$ vs. $(4yh)^{1/2}$:

Slope of this line is $C_v$
Determining the discharge $Q$

$$ Q = V_2 \cdot A_j = V_j \cdot A_j $$

We only know orifice opening area $A_0$ not $A_j$.

Due to vena contracta $d_j$ is smaller than $d_0$. Therefore, we need to estimate the velocity at cross-section 0 (or estimate the area at cross-section $j$).

Continuity equation:

$$ V_0 = V_j \cdot \frac{A_j}{A_0} \quad \Rightarrow \quad \text{define coefficient of contraction:} \quad C_c = \frac{A_j}{A_0} \quad \therefore V_0 = C_c V_j $$

$$ Q = V_0 A_0 = C_c \cdot V_j A_0 = C_c C_v \sqrt{2gh} A_0 = C_d A_0 \sqrt{2gh} $$

We know $V_j$ from previous page

$C_d = C_v C_c$ is determined empirically
During the experiment:

1) In the constant head experiment, we can simply calculate

\[ C_d = \frac{Q}{A_0 \sqrt{2gh}} \]

with \( A_0 = \frac{\pi}{4} d_0^2 \)

i.e., measure five different sets of \( h \) and \( Q \); get \( C_d \) from linear regression.

2) In the variable head experiment, \( h \) decreases in time. If variable head \( h(t) \) is taken into account, we need to consider mass conservation of the entire tank:

\[ A \frac{dh}{dt} = -C_d A_0 \sqrt{2gh} \]

\((A=160 \text{ cm}^2 \text{ is the cross-sectional area of the tank})\)
Given initial condition \( h(0) \) at \( t=0 \):

\[
\frac{dh}{\sqrt{h}} = -C_d \frac{A_0}{A} \sqrt{2g \, dt}
\]

\[
\Rightarrow 2\sqrt{h} = -C_d \frac{A_0}{A} \sqrt{2g \, t} + B \quad \text{Apply initial condition: } B = 2\sqrt{h(0)}
\]

\[
\Rightarrow \sqrt{h} = -\frac{C_d}{2} \frac{A_0}{A} \sqrt{2g \, t} + 2\sqrt{h(0)}
\]

\[
\therefore \sqrt{\frac{h}{h(0)}} = 1 - \frac{C_d}{\sqrt{2}} \frac{A_0}{A} \sqrt{\frac{g}{h(0)}} \, t
\]

\[
\frac{\sqrt{h}}{\sqrt{h(0)}}
\]

\[
\text{Slope of this line is } -\frac{C_d}{\sqrt{2}} \frac{A_0}{A} \sqrt{\frac{g}{h(0)}}
\]

\[
\Rightarrow C_d \text{ can be determined}
\]