Hydraulic Jump

Hydraulic jump can be seen downstream of a spillway during the release of water in reservoir.

Concrete bed

Strong velocity
Severe bed erosion

Sand bed

Concrete bed

Hydraulic jump

Weak velocity due to hydraulic jump

Sand bed
1. Measure flow discharge, upstream and downstream flow depth $h_1$ and $h_2$. Then, compare with theoretically predicted $h_2$ (eqn 5 in class note). Try this for several different discharge $Q$.

2. Get head loss $h_{L1}$ from energy conservation and measured data. Compare this value with theoretically derived head loss $h_L$ (eqn 9 in class note).
Basic Characteristics of Hydraulic Jump

Roller regime is similar to wave breaker—strong energy dissipation.

The upstream Froude number:

$$\text{Fr}_1 = \frac{V_1}{\sqrt{gh_1}} > 1$$

For supercritical flow (low flow depth, high flow velocity)

The downstream Froude number:

$$\text{Fr}_2 = \frac{V_2}{\sqrt{gh_2}} < 1$$

For subcritical flow (high flow depth, low flow velocity)
The upstream and downstream flow depth of a hydraulic jump are related. This relationship can be derived as (eqn 5 in the class note):

Conservation of mass:

\[ V_1 h_1 b = V_2 h_2 b \quad \Rightarrow \quad V_1 h_1 = V_2 h_2 \]
Conservation of momentum:

\[ \sum_{x} F = \sum_{\text{out}} \text{Momentum} - \sum_{\text{in}} \text{Momentum} \]

\[ \Rightarrow F_1 - F_2 = \rho Q (V_2 - V_1) \]

If locations (1) and (2) are sufficiently away from the roller, we can assume the pressure distribution is hydrostatic.

Recall: Force acting on a submerged surface is equal to pressure at centroid of the submerged area times area of submerged surface.

\[ F_1 = P_c h_1 b = \left( \frac{1}{2} \rho g h_1 \right) \cdot h_1 b = \frac{1}{2} \rho g h_1^2 b \]

\[ F_2 = \frac{1}{2} \rho g h_2^2 b \]

Momentum conservation

\[ \Rightarrow \frac{1}{2} \rho g b (h_1^2 - h_2^2) = \rho V_1 h_1 b (V_2 - V_1) \]

\[ \Rightarrow \frac{1}{2} g (h_1^2 - h_2^2) = V_1 h_1 (V_2 - V_1) \]
Replace $V_2$ by continuity, i.e., $V_2 = V_1 h_1 / h_2$

$$\Rightarrow \frac{1}{2} (h_1 - h_2)(h_1 + h_2) = \frac{V_1 h_1}{g} \frac{V_1}{h_2} (h_1 - h_2)$$

$$\Rightarrow (h_1 + h_2) = 2 Fr_1^2 \frac{h_1}{h_2}$$

$$\Rightarrow \frac{h_2}{h_1^2} (h_1 + h_2) = 2 Fr_1^2$$

$$\Rightarrow \left(\frac{h_2}{h_1}\right)^2 + \left(\frac{h_2}{h_1}\right) - 2 Fr_1^2 = 0$$

set: $Y = \frac{h_2}{h_1}$

We can easily find solution of this $2^{nd}$ order polynomial.
Discard negative root.

\[ i.e., \text{downstream flow depth of a hydraulic jump can be predicted by upstream flow information.} \]
Conservation of energy (to get head loss):  Note: $P_1=P_2=0$

$$h_1 + \frac{V_1^2}{2g} = h_2 + \frac{V_2^2}{2g} + h_L$$

$\implies$ You can directly get $h_{L1}$ from measured upstream & downstream flow depth, flow rate

$$h_L = (h_1 - h_2) + \frac{1}{2g} \left( V_1^2 - V_2^2 \right)$$

$$\frac{1}{2g} \left( V_1^2 - \frac{V_1^2 h_1^2}{h_2^2} \right) = \frac{V_1^2}{2g} \left( 1 - \frac{h_1^2}{h_2^2} \right) = \frac{V_1^2}{2g} h_1 \left( 1 - \frac{h_1^2}{h_2^2} \right) = \frac{1}{2} \text{Fr}_1^2 h_1 \left( 1 - \frac{h_1^2}{h_2^2} \right)$$

$\implies$ $h_L = (h_1 - h_2) + \frac{1}{2} \text{Fr}_1^2 h_1 \left( 1 - \frac{h_1^2}{h_2^2} \right)$

$$\frac{h_L}{h_1} = \left( 1 - \frac{h_2}{h_1} \right) + \frac{1}{2} \text{Fr}_1^2 \left[ 1 - \left( \frac{h_1}{h_2} \right)^2 \right]$$

Recall previously:  $\text{Fr}_1^2 = \frac{1}{2} \left[ \left( \frac{h_2}{h_1} \right)^2 + \left( \frac{h_2}{h_1} \right) \right]$

$Y = \frac{h_2}{h_1}$  $\therefore$  $\frac{1}{2} \text{Fr}_1^2 = \frac{1}{4} \left( Y^2 + Y \right)$
Hence,
\[
\frac{h_L}{h_1} = (1-Y) + \frac{1}{4}(Y^2 + Y)\left(1 - \frac{1}{Y^2}\right)
\]
\[
= \frac{1}{4Y}(Y^3 - 3Y^2 + 3Y - 1)
\]
\[
= \frac{(Y-1)^3}{4Y}
\]

Substitute $h_2, h_1$ back

\[
= \frac{(h_2 - h_1)^3}{4h_2h_1^2}
\]

\[
\therefore h_L = \frac{(h_2 - h_1)^3}{4h_2h_1}
\]

$\Rightarrow h_L$ obtained theoretically can be compared with directly measured value $h_{L1}$